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# Seismic Wave Propagation and Scattering in the Heterogeneous Earth

*Second Edition*

 Springer

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**SEISMIC WAVE  
PROPAGATION AND  
SCATTERING IN THE  
HETEROGENEOUS EARTH**

SECOND EDITION

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*Dedicated to*

*Keiiti Aki (1930-2005)*

*Inspired Mentor and Pioneer of Modern Seismology*

## Preface to the Second Edition

Scattering due to randomly distributed small-scale heterogeneities in the earth significantly changes seismic waveforms of local earthquakes especially for short periods. Scattering excites long lasting coda waves after the direct arrival and broadens the apparent duration of oscillation with increasing travel distance much longer than the source duration time. Models of propagation through deterministic structures such as those with horizontally uniform velocity layers cannot explain those observed phenomena. Our goal in writing this book is to put a focus on the phenomena of seismic wave scattering by distributed heterogeneities in the earth, especially in the lithosphere, where stochastic treatment is essential to describe both heterogeneous media and wave propagation through them. Stochastic approaches and deterministic approaches are complementary for the construction of a unified image of the earth's structure.

Keiiti Aki was a distinguished pioneer who extensively developed various stochastic methods for short-period seismology. His strong encouragement and continuous support for us were essential in motivating us to write the first edition of this book. Before Kei passed away in 2005, he kindly cited our book when he argued for the importance of the study on seismic wave scattering caused by small-scale heterogeneity in his letter to V. I. Keilis-Borok, "... To a geodynamicist, the earth's property is smoothly varying within bodies bounded by large-scale interfaces. Most seismologists also belong to this "smooth earth club," because once you start with an initial model of smooth earth your data usually do not require the addition of small-scale heterogeneity to your initial model. As summarized well in a recent book by *Sato and Fehler* [1998], the acceptance of coda waves in the data set is needed for the acceptance of small-scale seismic heterogeneity of the lithosphere. There are an increasing number of seismologists who accept it, forming the "rough earth club." I believe that you are also a member of the rough earth club, judging from the emphasis on the hierarchical heterogeneity of the lithosphere. ... " [Aki, 2009].

The first edition of this book was fortunately accepted in the geophysical community as a textbook, especially for graduate students. Furthermore it has been often cited in the physics community since this book introduced various aspects of wave scattering in real heterogeneous media. During the decade following the publication

of the first edition, there were developments in stochastic methods and analyses focusing on seismogram envelopes. Radiative transfer theory has been used not only for the study of coda envelopes but also for the analysis of whole seismogram envelopes. These studies made it possible to resolve the spatial variation of scattering strength. There have been developments in the statistical description of wave propagation in random media that reliably predict the delay of peak amplitude from the onset and the broadening of seismogram envelopes with increasing travel distance. Those methods have also been extended from scalar waves to vector waves. Investigators from throughout the world participated and collaborated in these developments as members of the IASPEI task group on “Scattering and Heterogeneity,” of which the summary was published in *Sato and Fehler* [2008].

In 2008, we started to write the second edition of this book. We expanded from the first edition by introducing recent developments in theory and analysis, updated illustrations and references, and wrote more precise steps in mathematical equations. The radiative transfer theory chapter and the Markov approximation chapter have been enlarged. We added two newly created chapters; one is a bridge between wave propagation in random media and the radiative transfer theory and the other one is on the Green’s function retrieval from the cross-correlation function of ambient noise.

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